GLOBAL ANALYSIS BASED STABILITY DESIGN OF STEEL STRUCTURES

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1. ABSTRACT

Ο έλεγχος ευστάθειας αποτελεί έναν από τους πιο σημαντικούς τύπους υπολογισμού των συνήθως λεπτών μεταλλικών κατασκευών, ο οποίος κυριαρχεί γενικά στον σχεδιασμό τους. Μέσω των παραδοσιακών μεθόδων σχεδιασμού έναντι ευστάθειας, τα μέλη μιας κατασκευής υπολογίζονται ως μεμονωμένα στοιχεία με κατάλληλες παραμέτρους (μήκη λυγισμού, μη στηριζόμενα μήκη, συντελεστές παρεμπόδισης κλπ) με σκοπό να ληφθεί υπόψη η συνέχεια με τα συνδεόμενα μέλη.
Στο κεφάλαιο 6.3.4 του Ευρωκώδικα ΕΝ 1993-1-1, ορίζεται μία καινοτόμος διαδικασία για το σχεδιασμό ευστάθειας των μεταλλικών κατασκευών με τη χρήση των αποτελεσμάτων της ανάλυσης ελαστικής ευστάθειας του καθολικού μοντέλου η οποία ονομάζεται συνήθως "Γενική μέθοδος". Η μέθοδος αυτή είναι βασικά κατάλληλη για χρήση μέσω λογισμικού, αλλά έχει κάποιες υπολογιστικές απαιτήσεις. Στο παρόν άρθρο, παρουσιάζεται η ορθή εφαρμογή της μεθόδου αυτής, σε σχέση με τους αριθμητικούς υπολογισμούς των παραμέτρων σχεδιασμού. Τα πιο σημαντικά πρακτικά προβλήματα στη διαδικασία αυτή είναι (1) οι ελάχιστες απαιτήσεις του μοντέλου προσομοίωσης για την αξιολόγηση της ελαστικής καθολικής μορφής λυγισμού και των ελαστικών κρίσιμων φορτίων και (2) η επιλογή της κατάλληλης μορφής λυγισμού για ένα μέλος του μοντέλου. Παράλληλα, εκτός από την αναφορά πρακτικών προτάσεων για τα ως άνω αναφερόμενα προβλήματα, παρουσιάζονται και κάποια παραδείγματα επαλήθευσης.

1. INTRODUCTION

When verifying the stability of beam-columns (members under combined axial load and bending) there are three different procedures in the current version of EN 1993-1-1 [1]:
(1) An imperfection approach described in Sections 5.2 and 5.3
(2) An isolated member approach described in Sections 6.3.1, 6.3.2 and 6.3.3
(3) The so-called “General method” (GM) described in Section 6.3.4
In the first approach the structural model is subjected to appropriate geometrical imperfections and after a completing a second order analysis only the cross section resistances need be checked (clause 5.2.2(7)(a)). This method is generally not used in practice due to the uncertainty in the definition of the shapes, amplitudes and signs of the equivalent imperfections. The second approach is the conventional engineering solution for

buckling problems, but is limited to uniform members only with relatively simple support and loading conditions. The method is based on two essential simplifications:

- Structural member isolation: the relevant member is isolated from the global structural model by applying special boundary conditions (supports, restraints or loads) at the connection points which are taken into account in the calculation of the buckling resistance.
- Buckling mode separation: the buckling of the member is calculated separately for the pure modes: flexural buckling for pure compression and lateral-torsional buckling for pure bending, and the two effects are connected by applying special interaction factors.

Although EN 1993-1-1 provides direction on the calculation of interaction factors in Annex A and Annex B, the choice of appropriate buckling lengths for complex problems is left entirely to the engineer.

The GM is a progressively new approach for stability design and only appeared late in the development of the Eurocodes – it did not appear in the draft of 1992, for example. The basic idea behind the GM is that it no longer isolates members and separates the pure buckling modes, but considers the complex system of forces in the member and evaluates the appropriate compound buckling modes. This calculation is usually done by direct global stability analysis of the whole structural model and normally suited for finite element analysis implemented into structural analysis software packages. The method offers the possibility to provide solutions where the isolated member approach is not entirely appropriate:

- It is applicable not only for single, isolated members but also for sub frames or complete structural models where the governing buckling mode involves the complete frame;
- It can examine irregular structural members such as tapered members, haunched members, and built-up members;
- It is applicable for any irregular load and support system where separation into the pure buckling modes is not possible.

Although in the current version of the Eurocode the GM is recommended only for lateral and lateral-torsional buckling of structural components, the basic approach may be extended to other cases. A number of research projects are underway across Europe intended to verify and widen its applicability [2].

2. DESCRIPTION OF THE "GENERAL METHOD"

The rules of the GM is defined in the Eurocode EN 1993-1-1 6.3.4. The GM uses the relevant global buckling modes and associated critical load factors for the out-of-plane stability verification of the structural model. The demonstrative example shows a simply supported HEA200 column restrained at mid-height laterally and torsionally (*Fig. 1 a*)). The column is subjected to compression and lateral uniformly distributed load acting eccentrically on the flange. The steps of the calculation of the buckling resistance (interaction of the lateral and lateral-torsional buckling) is shown in *Table 1* using both the classical isolated member approach (based on the separation of pure buckling modes) and the GM. In case of the GM an in-plane imperfection is added in order to include the second order amplification effect of the compression force on the major axis bending moment. All the necessary calculations are performed on ConSteel software [5].

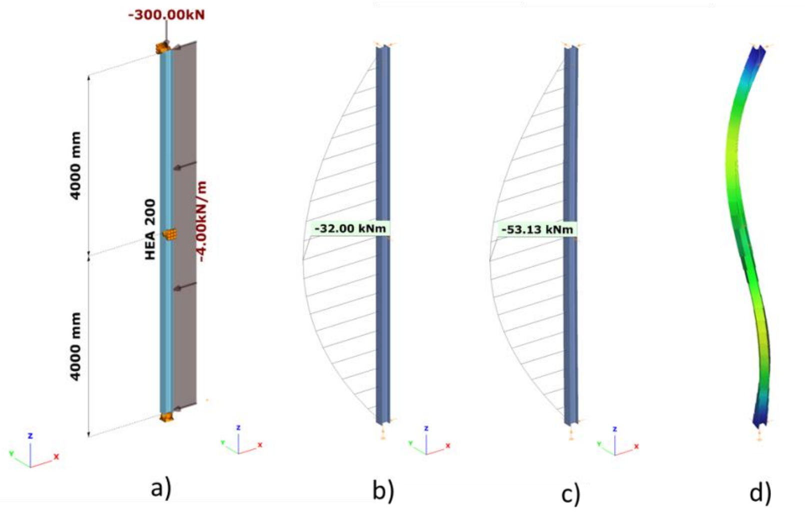

Fig. 1 a) Restrained column subjected to compression and bending, b) first order bending moment, c) second order bending moment, d) out-of-plane buckling shape

Steps		Classical approach EN 1993-1-1 6.3.3	"General method" EN 1993-1-1 6.3.4
1 a	*Imperfection: $e_{0y,d}$*	none	$e_{0y,d}$ *= 800/317 =2,53cm* [1b, Tab. NA.1]
1 b	*Member forces*	$N_{Ed} = 300kN$ $\max M_{y,Ed} = 32kNm$ (first order)	$N_{Ed} = 300kN$ $\max M_{y,Ed} = 53,1kNm$ (second order)
2	*Cross-section resistances*	$N_{c,Rk} = 1264,3kN$ (6.10)-(6.11) $M_{c,Rk} = 100,9kNm$ (6.13)-(6.15)	$\alpha_{ult,k} = \frac{1}{\frac{300}{1265}+\frac{53,1}{100,9}} = 1,310$ (6.65)
3	*Elastic critical forces and factors*	$N_{cr,y} = 1194kN$ § 6.3.1.2(1) $N_{cr,z} = 1736kN$ § 6.3.1.2(1) $M_{cr} = 220,9kNm$ § 6.3.2.2(2)	CONSTEEL: $\alpha_{cr,op} = 3,82$ § 6.3.4(3)
4	*Slenderness*	$\bar{\lambda}_y = \sqrt{\frac{N_{c,Rk}}{N_{cr,y}}} = 1,029$ § 6.3.1.2(1) $\bar{\lambda}_z = \sqrt{\frac{N_{c,Rk}}{N_{cr,z}}} = 0,855$ § 6.3.1.2(1) $\bar{\lambda}_{LT} = \sqrt{\frac{M_{c,Rk}}{M_{cr}}} = 0,676$ § 6.3.2.2(2)	$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = \sqrt{\frac{1,310}{3,82}} = 0,586$ (6.64)
5	*Reduction factors*	$\chi_y(\bar{\lambda}_y, b) = 0,580$ (6.49) $\chi_z(\bar{\lambda}_z, c) = 0,630$ (6.49) $\chi_{LT}(\bar{\lambda}_{LT}, b) = 0,880$ (6.57)	$\chi_z(\bar{\lambda}_{op}, c) = 0,794$ (6.49) $\chi_{LT}(\bar{\lambda}_{op}, b) = 0,924$ (6.57) $\chi_{op} = \min[\chi_z, \chi_{LT}] = 0,794$
6	*Interaction factors*	$k_{yy} = 1,292$ [1a, Annex B] $k_{zy} = 0,936$ [1a, Annex B]	–
7	*Stability check*	$\frac{N_{Ed}}{\frac{\chi_y N_{c,Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed}}{\frac{\chi_{LT} M_{c,Rk}}{\gamma_{M1}}} = 0,96$ (6.61) $\frac{N_{Ed}}{\frac{\chi_z N_{c,Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed}}{\frac{\chi_{LT} M_{c,Rk}}{\gamma_{M1}}} = 0,79$ (6.62)	$\frac{N_{Ed}}{\frac{\chi_z N_{c,Rk}}{\gamma_{M1}}} + \frac{M_{y,Ed}}{\frac{\chi_{LT} M_{c,Rk}}{\gamma_{M1}}} = 0,956$ (6.66) $\frac{N_{Ed}}{\frac{\chi_{op} N_{c,Rk}}{\gamma_{M1}}} + \frac{M_{y,Ed}}{\frac{\chi_{op} M_{c,Rk}}{\gamma_{M1}}} = 1,058$ (6.65)

Table 1. Steps of the "General method"

It can be seen that the basic difference is in the calculation of elastic critical forces (Step 3) where the integrated approach does not separate the pure loads but uses the complex system for the determination of the compound buckling mode (see *Fig. 1 d*) and the elastic critical load factor $\alpha_{cr,op}$ which is naturally includes all interactions between the different buckling effects. Accordingly one overall slenderness value describes the buckling problem and there is no need for interaction factors. The final utilization is quite similar to the result of the classical method. However since this methodology can be used in the same way for any type of loading and support system the uncertainties in the separation of the pure buckling modes and the determination of the necessary buckling parameters (buckling lengths, moment gradient factors and parameters in the interaction factors) are smartly eliminated. It is also noticeable that the method b) for the calculation of the reduction factors (formula 6.65) gives unnecessarily high utilization. One of the key points of the method is the calculation of the elastic critical load factor it is discussed in the next section.

3. EVALUATION OF THE ELASTIC CRITICAL LOAD FACTOR $\alpha_{cr,op}$

The power of the GM lies in the use of the complex elastic buckling analysis of the global structural model in order to evaluate the associated $\alpha_{cr,op}$ and the overall slenderness. There are more numerical FE model applicable for this buckling analysis however these should satisfy some mechanical aspects in order to be accurate and reliable:

- Cover all types of buckling modes – flexural, torsional, lateral-torsional, any interactions
- Cover the effect of member, load and support eccentricities
- Yield solution for member, load and support irregularities – web tapering, haunches, etc.

On the other hand from practical point of view the model should be not so complex to keep efficiency by the quick modelling and easy results handling, this is the *efficiency problem*. Satisfying both requirements the 7 DOF Vlasov beam element is proved to be a very accurate and efficient model for the global elastic buckling analysis [3] yielding reliable results for the buckling modes of steel structures. The elastic global stability analysis is usually performed by linear buckling analysis. In a standard finite element environment this problem can be expressed as a linear eigenvalue analysis with the following basic form:

$$(\mathbf{K}_E - \alpha \mathbf{K}_G)\mathbf{U} = 0 \tag{1}$$

where $\boldsymbol{K}_E$ is the elastic stiffness matrix, $\boldsymbol{K}_G$ is the second order geometric stiffness matrix, α is the eigenvalue and $\boldsymbol{U}$ is the corresponding eigenvector. In the mechanical interpretation the eigenvalue denotes the elastic critical load level and the eigenvector shows the eigenshape (eigenmode) or buckling shape (buckling mode).

As it has been shown the GM is evaluated on member level but the buckling modes are calculated on the global structural model. The correct application of the GM therefore requires the use of the most relevant buckling mode and the corresponding elastic critical load factor for the proper stability design of the member under examination. In the case of a complex 3D structural model with several load combinations and a great amount of different but relevant buckling modes it is usually not evident that for a certain member which is the most relevant mode for the design [4] this is the *relevancy problem*. This problem is quite complicated but also very significant, since in the case of a complex structural model it is usual, that different buckling modes describe the stability behavior of distinct parts of the model. For that reason a scaling procedure is necessary in order to select the appropriate buckling mode for the stability design of members. In order to do so the deformation energy generated by the i-th buckling mode is used as a basic measure which can be formulated as follows :

$$E_i = \frac{1}{2}\mathbf{U}_i^T \mathbf{K}_E \mathbf{U}_i \tag{2}$$

This deformation energy can be calculated for each single member k of the model from the same global buckling mode using the proper stiffness of matrix part k:

$$E_i^k = \frac{1}{2}\mathbf{U}_i^T \mathbf{K}_E^k \mathbf{U}_i \tag{3}$$

where the following summations holds for the global model composed of a total number of m members:

$$\mathbf{K}_E = \sum_{k=1}^{m} \mathbf{K}_E^k \text{ and } E_i = \sum_{k=1}^{m} E_i^k \tag{4}$$

Using these measures a specific scaling procedure can be constructed defining a so called *mode relevance factor* (*MRF*) which indicates what the relevant (critical) members (k) are for the i-th buckling mode. The basic assumption for this factor is that each buckling mode has one (or more) specific member(s) which is (are) the most critical and all the members are compared to this one to assess the contribution to the buckling:

$$MRF_i^k = 100 \frac{\mathbf{U}_i^T \mathbf{K}_E^k \mathbf{U}_i}{\max(\mathbf{U}_i^T \mathbf{K}_E^k \mathbf{U}_i)} [\%] \tag{5}$$

For the most critical member this factor always takes 100%, and the more critical a member the closer is the *MRF* to 100%. This factor can provide informative help for the engineer to select the most relevant buckling mode for the stability design of members in the complex 3D model.

4. EXAMPLE FOR THE USE OF THE GM

A 2D frame is presented as an example taken from the book of [5] at Section 9.9.5 page 413 and the geometry and loads modeled in ConSteel [5] are shown in *Fig. 2*.

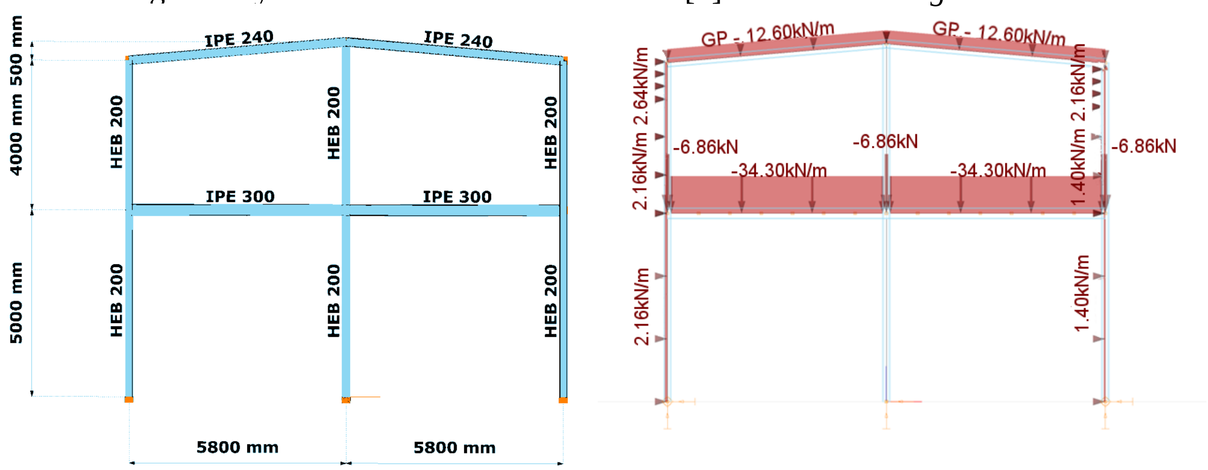

Fig. 2 Example of a two-bay two-storey frame

The two outside columns are fixed the inside column is pinned and the middle beam is considered to be prevented from any type of buckling by the connected slab. *Fig. 3* shows the first three dominant global buckling modes with the corresponding mode relevance factors for each member calculated by ConSteel [5].

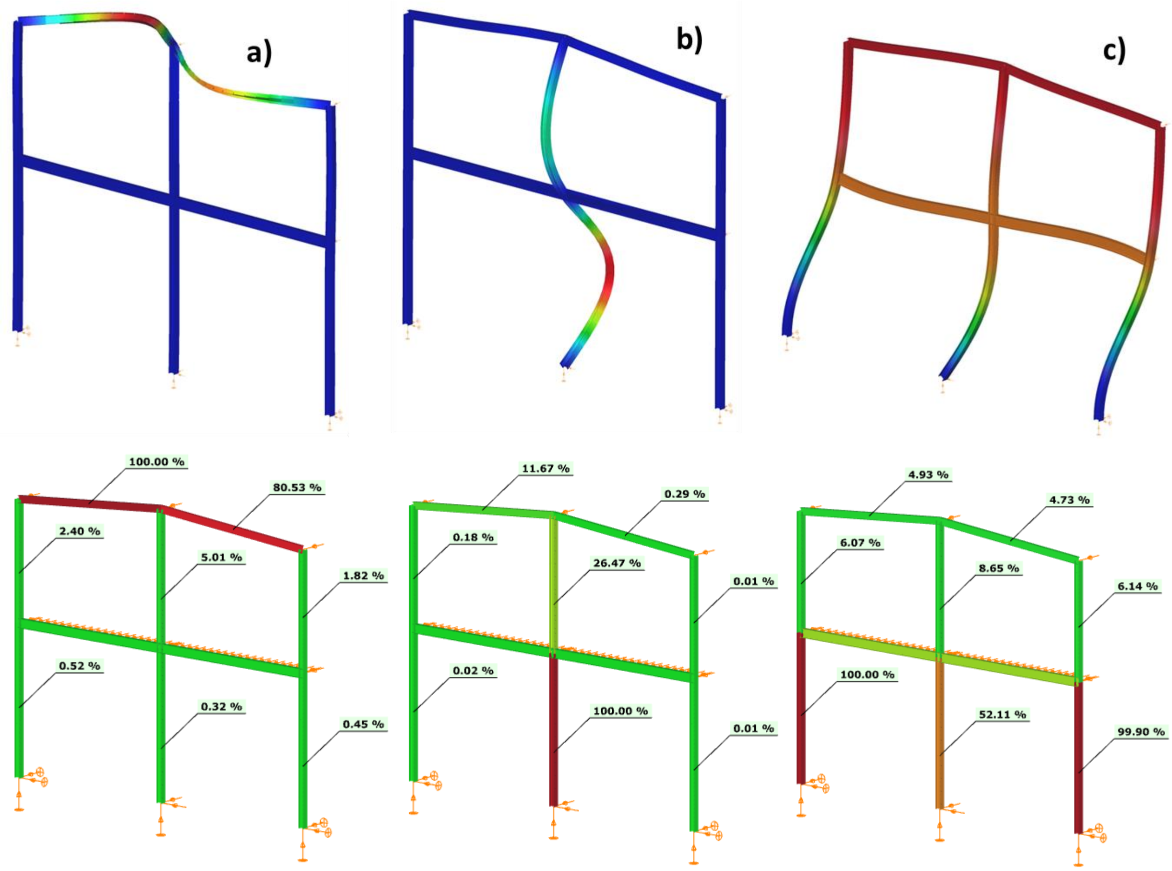

Fig. 3 Dominant global buckling modes and MRFs with elastic critical load factors a) $\alpha_{cr,op}$=2,63; b) $\alpha_{cr,op}$=7,88; b) α_{cr}=12,78

Studying the illustrated buckling modes it can be seen, that the first mode is the lateral-torsional buckling mode of the upper beam, the second mode is the flexural-torsional buckling of the middle column and the third one is the in-plane swaying buckling mode of the whole model. The last mode can not be the base of the GM based buckling design, since it is only valid for out-of-plane buckling, but can be applied as in-plane imperfection. The MRFs show apparently the critical members for the different buckling modes – in this demonstration example the validity of these factors can be easily checked by the graphic of the buckling modes. As it can be seen the MRFs are a very good measure for detecting the relevant buckling mode for a certain member which can support a highly automated and efficient stability design procedure together with the GM. Therefore for the upper beams the first mode is selected with $\alpha_{cr,op}$=2,63, for the middle column the second mode with $\alpha_{cr,op}$=7,88 and the member slenderness values are calculated accordingly. For the outside columns the third mode is used by applying as the proper equivalent geometric in-plane imperfection. In *Fig. 3* the contribution from other members can be also detected by the MRFs which is a valuable information on how isolated are the different buckling modes. In *Table 2* the results of the stability design checks based on the GM for the upper beam and middle column are illustrated and compared with the results of [5] using the classical design method. The final utilization values are quite comparable but the differences indicate the inaccuracy of the member isolation method: in case of the beams it yields lower utilization while for the columns the utilization is higher.

	Upper beam	**Middle column**
Dominant place of calculation	left beam, right end section	bottom column, uppermost section
N_{Ed} [kN]	-13,5	-282,2
$M_{y,Ed}$ [kNm]	33,2	17,5
$\alpha_{ult,k}$	2,04	3,37
$\alpha_{cr,op}$	2,63	7,88
$\bar{\lambda}_{op}$	0,881	0,654
χ	0,673	0,753
χ_{LT}	0,771	0,892
Utilization	**57,4%**	**36,8%**
Utilization in [40]	*52%*	*42%*

Table 2. Stability design checks of the frame based on the GM

5. CONCLUSIONS

The paper presented the description and application of the “General Method” which has been introduced by the EC3-1-1 6.3.4 for the stability design of steel structures. The basic rules and application steps are introduced and compared to the classical member isolation method in order to understand the different parameters. It is shown that the most fundamental step is the calculation of the elastic critical load factor for the overall slenderness of the members. A method for the selection of relevant buckling mode is presented and shown on a demonstrative example.

6. REFERENCES

[1] European Standard, EuroCode 3. *Design of Steel Structures – Part1-1: General rules and rules for buildings*, EN 1993-1-1, 2005.

[2] Papp F, Rubert A, Szalai J. “DIN EN 1993-1-1-konforme integrierte Stabilitätsanalysen für 2D/3D-Stahlkonstruktionen (Teil 1-3)“. *Stahlbau* 2014; 83: Heft 1, 2, 5.

[3] Papp F, Szalai J. “Theory and application of the General Method“ *EUROSTEEL* 2011, Budapest, Hungary

[4] Szalai J. “Use of eigenvalue analysis for different levels of stability design” *Stability and Ductility of Steel Structures*, SDSS, 2010, Rio de Janeiro, Brazil

[5] ConSteel Structural Analysis and Design Software, ConSteel Solutions Ltd. Hungary, www.consteelsoftware.com

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SUMMARY

The stability check is one of the most basic design type of the usually slender steel structures which generally govern the design. In the traditional way of stability design the members of the structure are calculated as single isolated elements with proper parameters (buckling lengths, unbraced lengths, end restrain factors etc.) in order to consider the connectivity to the surrounding structural members. The Eurocode EN 1993-1-1 in section 6.3.4 defines a new and innovative procedure for the stability design of steel structures using the results of the elastic stability analysis of the global structural model, it is usually called “General method”. The method is basically suited for software, but has some requirements for the calculations needed. In this paper the correct application is reviewed regarding the numerical calculation of the design parameters. The most important practical problems are (1) the minimum requirements of the analysis model for the evaluation of the elastic global buckling modes and elastic critical loads and (2) the selection of the proper buckling mode for a member of the model. Besides giving practical proposals for the problems some validation examples are also presented.